

# Kaon and antikaon optical potentials in isospin asymmetric hyperonic matter

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The medium modifications of the energies of kaons and antikaons in isospin asymmetric hyperonic matter are investigated using a chiral SU(3) model. The isospin dependent medium effects, are important for asymmetric heavy ion collision experiments, as well as relevant for the neutron star phenomenology as the bulk matter in the interior of the neutron star is highly isospin asymmetric. The effects of hyperons on the medium modifications of the kaons and antikaons in the strange hadronic matter are investigated in the present work and are seen to be appreciable for hadronic matter with large strangeness fractions. The study of the K-mesons in the asymmetric strange hadronic matter can be especially relevant for the compressed strange baryonic matter which can result from asymmetric heavy ion collision experiments in the future accelerator facility FAIR at GSI.

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## I. INTRODUCTION

The topic of study of the in-medium properties of hadrons is an important problem in strong interaction physics, which has relevance in the high energy heavy-ion collision experiments, as well as in neutron star phenomenology. In heavy ion collision experiments, the medium modifications of the hadrons can be seen in different observables, like particle yield, particle spectra as well as in their collective flow. The study of medium modifications of K-mesons were initiated by Kaplan and Nelson [1], who suggested the possibility of antikaon condensation in the interior of the neutron stars due to the drop in the mass of the antikaons in the nuclear medium. However, recent experimental observations on neutron star phenomenology impose constraints on the nuclear equation of state (EOS). The EOS for the nuclear matter obtained using an effective model should be consistent with the astrophysical bounds to be acceptable as an EOS for neutron star matter [2, 3]. Recently, the nuclear matter EOS have been investigated consistent with the neutron star phenomenology as well as data for collective flow in heavy ion collision experiments [4]. The in-medium modification of kaon/antikaon properties can be observed experimentally [5, 6, 7, 8, 9] in relativistic nuclear collisions from their abundance, spectra as well as collective flow. There have also been intense theoretical investigations [10, 11, 12, 13, 14, 15, 16, 17, 18, 19] on the kaon and antikaon properties in the dense and hot hadronic matter, and to study the effects of such modifications on the observables of the high energy heavy ion collision experiments. Hence there have been extensive research to obtain the in-medium properties of kaons and antikaons due to their relevance for the high energy heavy ion collision experiments. These have been calculated by using several methods like extracting from kaonic atom data, using chiral lagrangians or by coupled channel methods [20]. However, the kaon/antikaon potential in the hot and dense hadronic matter remains still an unresolved issue.

The isospin effects in hot and dense hadronic matter [22] are important in isospin asymmetric heavy-ion collision experiments. Within the UrQMD model the density dependence of the symmetry potential has been studied by investigating observables like the  $\pi^-/\pi^+$  ratio, the n/p ratio, the  $\Delta^-/\Delta^{++}$  ratio, NN scattering cross section and pion flow for neutron rich heavy ion collisions [23]. Within this UrQMD model, the isospin effects on the production of  $K^0$  and  $K^+$  [24] have also been studied.

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In [21] it was pointed out that one has to take into account the effect of the Haar measure in mean field approximations of non-linear chiral models, which is especially relevant in the high-temperature regime. An extended discussion of the model presented here, including this contribution should be performed in future studies.

In the present investigation we use a chiral  $SU(3)$  model [25, 26] for the description of hadrons in the medium. The model has been used to study nuclear matter, finite nuclei, and hyperonic matter. The properties of the vector mesons have also been investigated in the nuclear medium within this model [26, 27]. The energies of kaons (antikaons) in the (asymmetric) nuclear matter [28, 29, 30] were also studied within this framework and the isospin dependence was seen to be appreciable for high densities and hence will be particularly relevant in the future compressed baryonic matter (CBM) experiment at FAIR, GSI. The kaon and antikaon properties have been investigated consistent with the low energy KN scattering data [31, 32]. In the present work, we investigate the effect of strangeness on the kaon and antikaon optical potentials in the isospin asymmetric hyperonic matter, consistent with the low energy kaon nucleon scattering lengths for channels  $I=0$  and  $I=1$  [29, 30].

The outline of the paper is as follows: In section II we shall briefly review the  $SU(3)$  model used in the present investigation. Section III describes the medium modification of the  $K(\bar{K})$  mesons in this effective chiral model. In section IV, we discuss the results obtained for the optical potentials of the kaons and antikaons and the isospin-dependent effects on these optical potentials in asymmetric hyperonic matter. Section V summarizes our results and discusses possible extensions of the calculations.

## II. THE HADRONIC CHIRAL $SU(3) \times SU(3)$ MODEL

The effective hadronic chiral Lagrangian used in the present work is given as

$$\mathcal{L} = \mathcal{L}_{kin} + \sum_{W=X,Y,V,A,u} \mathcal{L}_{BW} + \mathcal{L}_{vec} + \mathcal{L}_0 + \mathcal{L}_{SB} \quad (1)$$

are discussed. Eq. (1) corresponds to a relativistic quantum field theoretical model of baryons and mesons adopting a nonlinear realization of chiral symmetry [33, 34, 35] and broken scale invariance (for details see [25, 26, 27]) as a description of the hadronic matter. The model was used successfully to describe nuclear matter, finite nuclei, hypernuclei and neutron stars. The Lagrangian contains the baryon octet, the spin-0 and spin-1 meson multiplets as the elementary degrees of freedom. In Eq. (1),  $\mathcal{L}_{kin}$  is the kinetic energy term,  $\mathcal{L}_{BW}$  contains the baryon-meson interactions in which the baryon-spin-0 meson interaction terms generate the baryon masses.  $\mathcal{L}_{vec}$  describes the dynamical mass generation of the vector mesons via couplings to the scalar fields and contains additionally quartic self-interactions of the vector fields.  $\mathcal{L}_0$  contains the meson-meson interaction terms inducing the spontaneous breaking of chiral symmetry as well as a scale invariance breaking logarithmic potential.  $\mathcal{L}_{SB}$  describes the explicit chiral symmetry breaking.

The baryon-scalar meson interactions generate the baryon masses and the parameters corresponding to these interactions are adjusted so as to obtain the baryon masses as their experimentally measured vacuum values. For the baryon-vector meson interaction terms, there exist the  $F$ -type (antisymmetric) and  $D$ -type (symmetric) couplings. Here we will use the antisymmetric coupling [25, 29] because, following the universality principle [36] and the vector meson dominance model, one can conclude that the symmetric coupling should be small. Additionally we choose the parameters [25, 29] so as to decouple the strange vector field  $\phi_\mu \sim \bar{s}\gamma_\mu s$  from the nucleon, corresponding to an ideal mixing between  $\omega$  and  $\phi$ . A small deviation of the mixing angle from the ideal mixing [37, 38, 39] has not been taken into account in the present investigation.

The Lagrangian densities corresponding to the interaction for the vector meson,  $\mathcal{L}_{vec}$ , the meson-meson interaction  $\mathcal{L}_0$  and that corresponding to the explicit chiral symmetry breaking  $\mathcal{L}_{SB}$  have been described in detail in references [25, 29].

To investigate the hadronic properties in the medium, we write the Lagrangian density within the chiral  $SU(3)$  model in the mean field approximation and determine the expectation values of the meson fields by minimizing the thermodynamical potential [26, 27].

## III. KAON (ANTIKAON) INTERACTIONS IN THE CHIRAL $SU(3)$ MODEL

In this section, we derive the dispersion relations for the  $K(\bar{K})$  [40] and calculate their optical potentials in asymmetric [29, 30] hadronic matter, taking into the effects from hyperons. The modifications of the kaon and antikaon

energies arise due to the effects from scalar mesons, scalar-isovector  $\delta$  meson and due to interactions with the nucleons and hyperons. The interactions of kaons and antikaons with the baryon octet is due to the vectorial Weinberg-Tomazawa as well as due to terms similar to the range term in chiral perturbation theory. It might be noted here that the interaction of the pseudoscalar mesons to the vector mesons, in addition to the pseudoscalar meson–nucleon vectorial interaction leads to a double counting in the linear realization of the chiral effective theory [41]. Within the nonlinear realization of the chiral effective theories, such an interaction does not arise in the leading or sub-leading order, but only as a higher order contribution [41]. Hence the vector meson-pseudoscalar interaction will not be considered within the present investigation. In the following, we shall derive the dispersion relations for the kaons and antikaons and study the dependence of the kaon and antikaon optical potentials on the isospin asymmetric parameter,  $\eta = \frac{1}{2}(\rho_n - \rho_p)/\rho_B$ . For this, we shall include the effects from isospin asymmetry originating from the scalar-isovector  $\delta$  field, vectorial interaction with the nucleons, an isospin symmetric range term [29] as well as an isospin dependent range term arising from the interaction with the nucleons [30]. In the present investigation, we include the effects from the hyperons to study the kaon and antikaon properties, which were not taken into account in the earlier works.

The interaction Lagrangian modifying the energies of the  $K(\bar{K})$ -mesons is given as

$$\begin{aligned}
\mathcal{L}_{KN} = & -\frac{i}{4f_K^2} \left[ \left( 2\bar{p}\gamma^\mu p + \bar{n}\gamma^\mu n - \bar{\Sigma}^-\gamma^\mu \Sigma^- + \bar{\Sigma}^+\gamma^\mu \Sigma^+ - 2\bar{\Xi}^-\gamma^\mu \Xi^- - \bar{\Xi}^0\gamma^\mu \Xi^0 \right) \right. \\
& \times \left( K^-(\partial_\mu K^+) - (\partial_\mu K^-)K^+ \right) \\
& + \left( \bar{p}\gamma^\mu p + 2\bar{n}\gamma^\mu n + \bar{\Sigma}^-\gamma^\mu \Sigma^- - \bar{\Sigma}^+\gamma^\mu \Sigma^+ - \bar{\Xi}^-\gamma^\mu \Xi^- - 2\bar{\Xi}^0\gamma^\mu \Xi^0 \right) \\
& \times \left( \bar{K}^0(\partial_\mu K^0) - (\partial_\mu \bar{K}^0)K^0 \right) \Big] \\
& + \frac{m_K^2}{2f_K} \left[ (\sigma + \sqrt{2}\zeta + \delta)(K^+K^-) + (\sigma + \sqrt{2}\zeta - \delta)(K^0\bar{K}^0) \right] \\
& - \frac{1}{f_K} \left[ (\sigma + \sqrt{2}\zeta + \delta)(\partial_\mu K^+)(\partial^\mu K^-) + (\sigma + \sqrt{2}\zeta - \delta)(\partial_\mu K^0)(\partial^\mu \bar{K}^0) \right] \\
& + \frac{d_1}{2f_K^2} (\bar{p}p + \bar{n}n + \bar{\Lambda}^0\Lambda^0 + \bar{\Sigma}^+\Sigma^+ + \bar{\Sigma}^0\Sigma^0 + \bar{\Sigma}^-\Sigma^- + \bar{\Xi}^-\Xi^- + \bar{\Xi}^0\Xi^0) \\
& \times ((\partial_\mu K^+)(\partial^\mu K^-) + (\partial_\mu K^0)(\partial^\mu \bar{K}^0)) \\
& + \frac{d_2}{2f_K^2} \left[ (\bar{p}p + \frac{5}{6}\bar{\Lambda}^0\Lambda^0 + \frac{1}{2}\bar{\Sigma}^0\Sigma^0 + \bar{\Sigma}^+\Sigma^+ + \bar{\Xi}^-\Xi^- + \bar{\Xi}^0\Xi^0)(\partial_\mu K^+)(\partial^\mu K^-) \right. \\
& \left. + (\bar{n}n + \frac{5}{6}\bar{\Lambda}^0\Lambda^0 + \frac{1}{2}\bar{\Sigma}^0\Sigma^0 + \bar{\Sigma}^-\Sigma^- + \bar{\Xi}^-\Xi^- + \bar{\Xi}^0\Xi^0)(\partial_\mu K^0)(\partial^\mu \bar{K}^0) \right]
\end{aligned} \tag{2}$$

In (2) the first term is the vectorial interaction term (Weinberg-Tomozawa term) obtained from the kinetic term of the Lagrangian, the second and third terms are obtained from the explicit symmetry breaking and the pseudoscalar kinetic terms of the chiral effective Lagrangian respectively [29, 30]. The fourth and fifth terms in (2) for the KN interactions arise from the terms

$$\mathcal{L}_{(d_1)}^{BM} = \frac{d_1}{2} Tr(u_\mu u^\mu) Tr(\bar{B}B), \tag{3}$$

and,

$$\mathcal{L}_{(d_2)}^{BM} = d_2 Tr(\bar{B}u_\mu u^\mu B). \tag{4}$$

in the SU(3) chiral model [28, 29]. The last three terms in (2) represent the range term in the chiral model, with the last term being an isospin asymmetric interaction. We might note here that in equation (2), we have not written the terms which are of the form  $\bar{B}_i B_j$ , with  $i \neq j$ . These types of terms are however relevant for the calculation of the kaon-nucleon scattering terms [30]. The Fourier transformation of the equation-of-motion for kaons (antikaons) leads to the dispersion relations,

$$-\omega^2 + \vec{k}^2 + m_K^2 - \Pi(\omega, |\vec{k}|) = 0,$$

where  $\Pi$  denotes the kaon (antikaon) self energy in the medium.

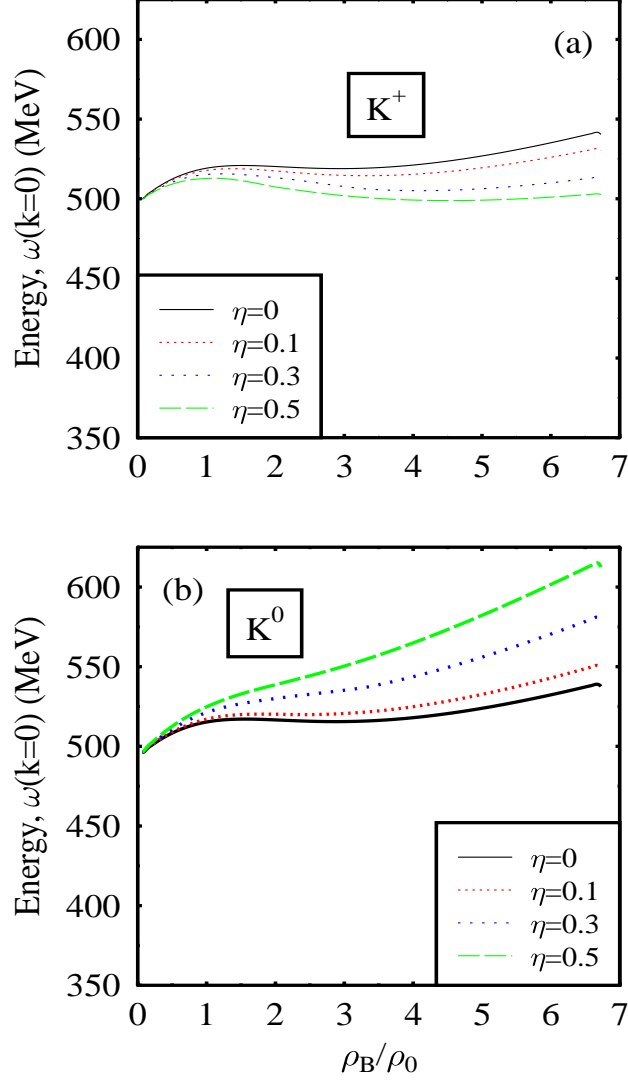


FIG. 1: The kaon energies (for  $K^+$  in (a) and for  $K^0$  in (b)) in MeV plotted as a functions of the baryon density,  $\rho_B/\rho_0$  for  $f_s=0.1$  and for different values of the isospin asymmetry parameter,  $\eta = \frac{1}{2}(\rho_n - \rho_p)/\rho_B$ .

Explicitly, the self energy  $\Pi(\omega, |\vec{k}|)$  for the kaon doublet,  $(K^+, K^0)$  arising from the interaction (2) is given as

$$\begin{aligned}
 \Pi(\omega, |\vec{k}|) = & -\frac{1}{4f_K^2} \left[ 3(\rho_p + \rho_n) \pm (\rho_p - \rho_n) \pm 2(\rho_{\Sigma^+} - \rho_{\Sigma^-}) - (3(\rho_{\Xi^-} + \rho_{\Xi^0}) \pm (\rho_{\Xi^-} - \rho_{\Xi^0})) \right] \omega \\
 & + \frac{m_K^2}{2f_K} (\sigma' + \sqrt{2}\zeta' \pm \delta') \\
 & + \left[ -\frac{1}{f_K} (\sigma' + \sqrt{2}\zeta' \pm \delta') + \frac{d_1}{2f_K^2} (\rho_s^p + \rho_s^n + \rho_s^{\Lambda^0} + \rho_s^{\Sigma^+} + \rho_s^{\Sigma^0} + \rho_s^{\Sigma^-} + \rho_s^{\Xi^-} + \rho_s^{\Xi^0}) \right. \\
 & \left. + \frac{d_2}{4f_K^2} \left( (\rho_s^p + \rho_s^n) \pm (\rho_s^p - \rho_s^n) + \rho_s^{\Sigma^0} + \frac{5}{3}\rho_s^{\Lambda^0} + (\rho_s^{\Sigma^+} + \rho_s^{\Sigma^-}) \pm (\rho_s^{\Sigma^+} - \rho_s^{\Sigma^-}) \right) \right]
 \end{aligned}$$

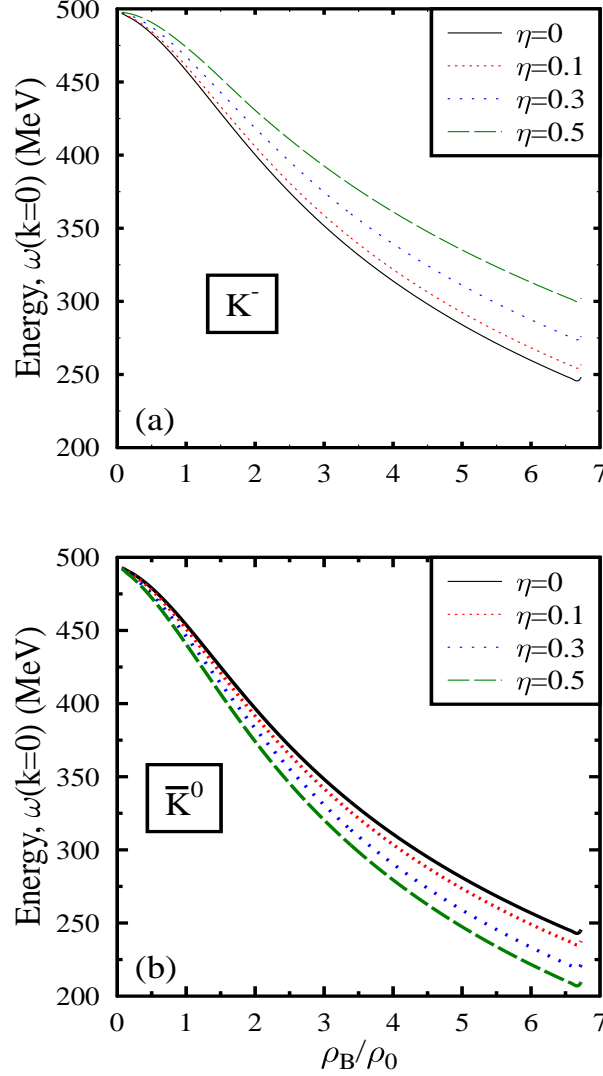


FIG. 2: The energies of the antikaons (for  $K^-$  in (a) and for  $\bar{K}^0$  in (b)), at zero momentum as functions of the baryon density ( $\rho_B/\rho_0$ ), are plotted for  $f_s=0.1$  and for different values of the isospin asymmetry parameter,  $\eta$ .

$$+ 2\rho_{\Xi^-}^s + 2\rho_{\Xi^0}^s \Big] (\omega^2 - \vec{k}^2), \quad (5)$$

where the  $\pm$  signs refer to the  $K^+$  and  $K^0$  respectively. In the above,  $\sigma' (= \sigma - \sigma_0)$ ,  $\zeta' (= \zeta - \zeta_0)$  and  $\delta' (= \delta - \delta_0)$  are the fluctuations of the scalar-isoscalar fields  $\sigma$  and  $\zeta$ , and the third component of the scalar-isovector field,  $\delta$ , from their vacuum expectation values. The vacuum expectation value of  $\delta$  is zero ( $\delta_0=0$ ), since a nonzero value for it will break the isospin symmetry of the vacuum (the small isospin breaking effect coming from the mass and charge difference of the up and down quarks has been neglected here).  $\rho_i$  and  $\rho_i^s$  with  $i = p, n, \Lambda, \Sigma^\pm, \Sigma^0, \Xi^-, \Xi^0$  are the number density and the scalar density of the baryon of type  $i$ .

Similarly, for the antikaon doublet,  $(K^-, \bar{K}^0)$ , the self-energy is calculated as

$$\Pi(\omega, |\vec{k}|) = \frac{1}{4f_K^2} \left[ 3(\rho_p + \rho_n) \pm (\rho_p - \rho_n) \pm 2(\rho_{\Sigma^+} - \rho_{\Sigma^-}) - (3(\rho_{\Xi^-} + \rho_{\Xi^0}) \pm (\rho_{\Xi^-} - \rho_{\Xi^0})) \right] \omega$$

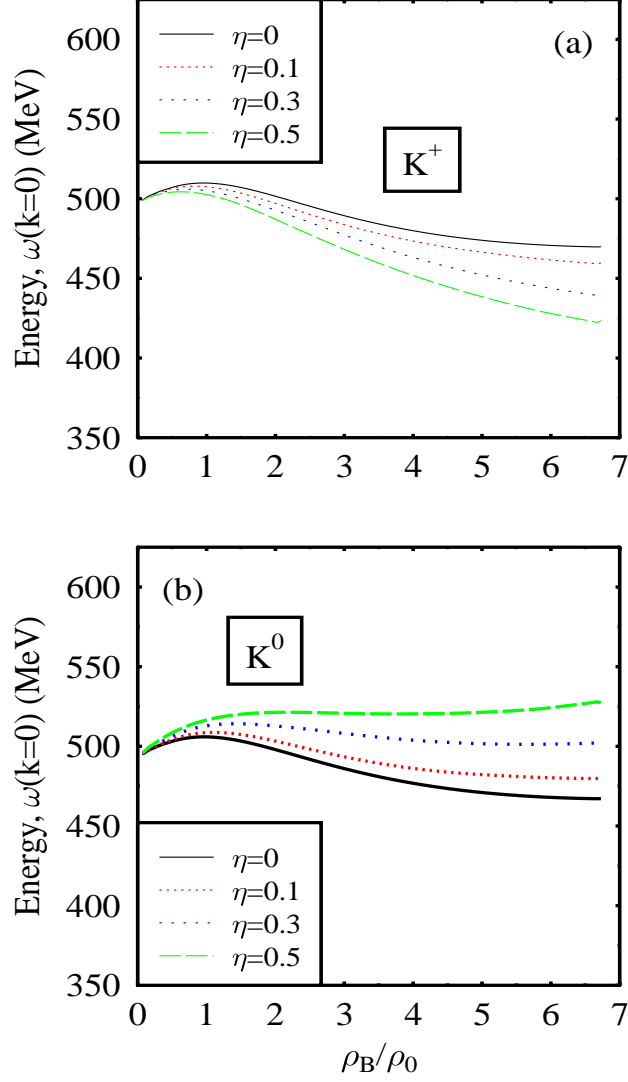


FIG. 3: The kaon energies (for  $K^+$  in (a) and for  $K^0$  in (b)) in MeV plotted as a functions of the baryon density,  $\rho_B/\rho_0$  for  $f_s=0.3$  and for different values of the isospin asymmetry parameter,  $\eta = \frac{1}{2}(\rho_n - \rho_p)/\rho_B$ .

$$\begin{aligned}
& + \frac{m_K^2}{2f_K}(\sigma' + \sqrt{2}\zeta' \pm \delta') \\
& + \left[ -\frac{1}{f_K}(\sigma' + \sqrt{2}\zeta' \pm \delta') + \frac{d_1}{2f_K^2}(\rho_s^p + \rho_s^n + \rho_s^{\Lambda^0} + \rho_s^{\Sigma^+} + \rho_s^{\Sigma^0} + \rho_s^{\Sigma^-} + \rho_s^{\Xi^-} + \rho_s^{\Xi^0}) \right. \\
& + \frac{d_2}{4f_K^2} \left( (\rho_s^p + \rho_s^n) \pm (\rho_s^p - \rho_s^n) + \rho_s^{\Sigma^0} + \frac{5}{3}\rho_s^{\Lambda^0} + (\rho_s^{\Sigma^+} + \rho_s^{\Sigma^-}) \pm (\rho_s^{\Sigma^+} - \rho_s^{\Sigma^-}) \right. \\
& \left. \left. + 2\rho_s^{\Xi^-} + 2\rho_s^{\Xi^0} \right) \right] (\omega^2 - \vec{k}^2), \tag{6}
\end{aligned}$$

where the  $\pm$  signs refer to the  $K^-$  and  $\bar{K}^0$  respectively.

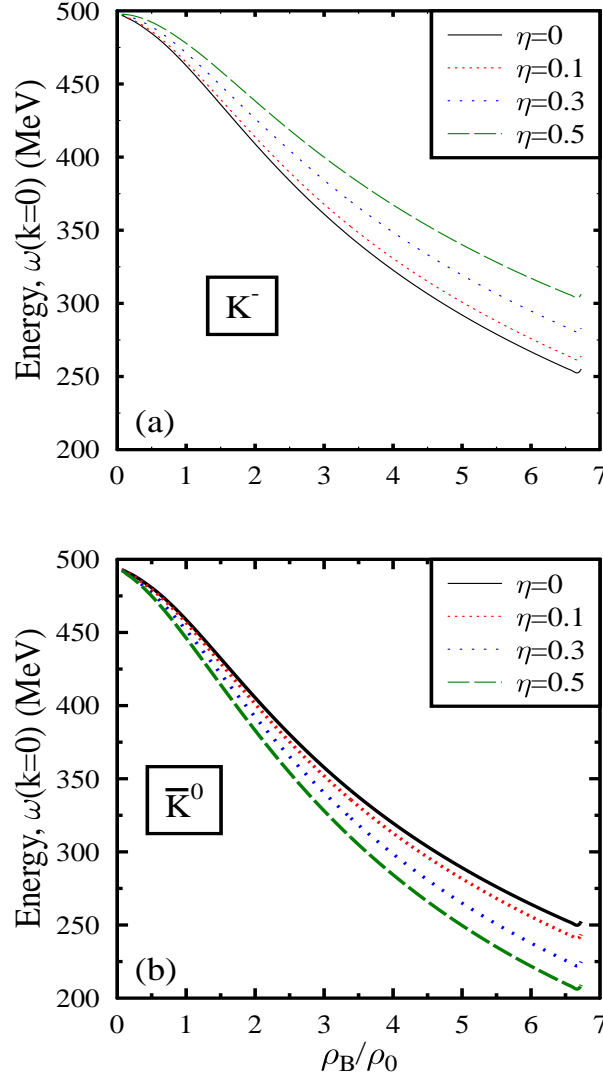


FIG. 4: The energies of the antikaons (for  $K^-$  in (a) and for  $\bar{K}^0$  in (b)), at zero momentum as functions of the baryon density ( $\rho_B/\rho_0$ ), are plotted for  $f_s=0.3$  and for different values of the isospin asymmetry parameter,  $\eta$ .

The optical potentials are calculated from the energies of the kaons and antikaons using

$$U(\omega, k) = \omega(k) - \sqrt{k^2 + m_K^2}, \quad (7)$$

where  $m_K$  is the vacuum mass for the kaon (antikaon).

The parameters  $d_1$  and  $d_2$  are calculated from the empirical values of the KN scattering lengths for  $I=0$  and  $I=1$  channels [29], which are taken as [31, 42, 43]

$$a_{KN}(I=0) \approx -0.09 \text{ fm}, \quad a_{KN}(I=1) \approx -0.31 \text{ fm}. \quad (8)$$

leading to the isospin averaged KN scattering length as

$$\bar{a}_{KN} = \frac{1}{4}a_{KN}(I=0) + \frac{3}{4}a_{KN}(I=1) \approx -0.255 \text{ fm}. \quad (9)$$

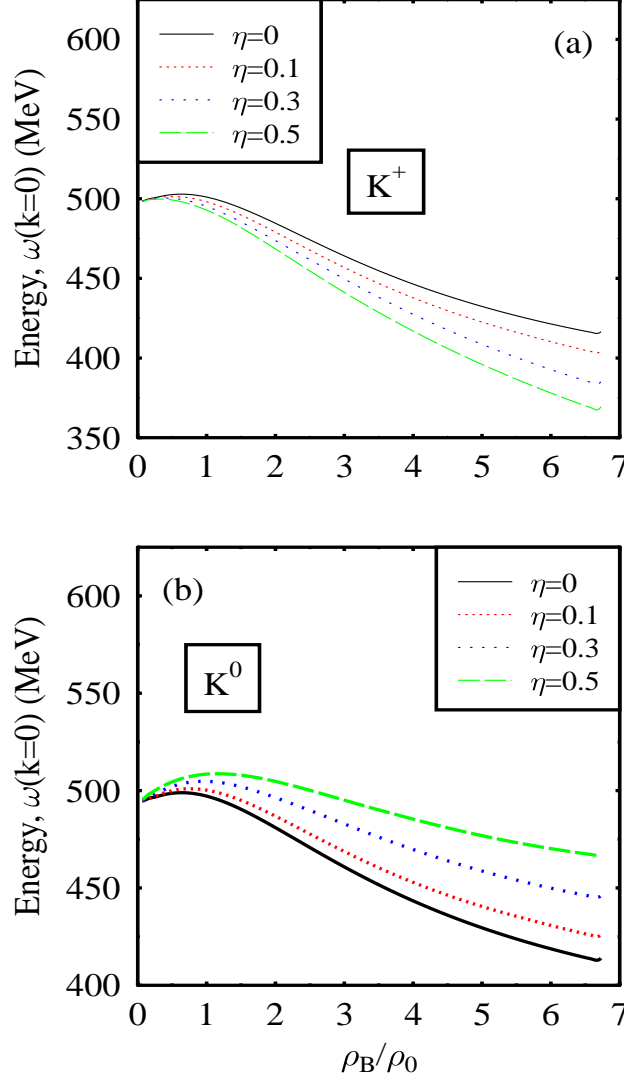


FIG. 5: The kaon energies (for  $K^+$  in (a) and for  $K^0$  in (b)) in MeV plotted as a functions of the baryon density,  $\rho_B/\rho_0$  for  $f_s=0.5$  and for different values of the isospin asymmetry parameter,  $\eta = \frac{1}{2}(\rho_n - \rho_p)/\rho_B$ .

#### IV. RESULTS AND DISCUSSIONS

The present calculations use the following model parameters. The values,  $g_{\sigma N} = 10.6$ , and  $g_{\zeta N} = -0.47$  are determined by fitting vacuum baryon masses. The other parameters as fitted to the asymmetric nuclear matter saturation properties in the mean field approximation are:  $g_{\omega N}=13.3$ ,  $g_{\rho N}=5.5$ ,  $g_4=79.7$ ,  $g_{N\delta}=2.5$ ,  $m_\zeta=1024.5$  MeV,  $m_\sigma=466.5$  MeV and  $m_\delta=899.5$  MeV [30]. The coefficients  $d_1$  and  $d_2$ , calculated from the empirical values of the KN scattering lengths for I=0 and I=1 channels (8), are  $2.56/m_K$  and  $0.73/m_K$  respectively.

The kaon and antikaon properties were studied in the isospin symmetric nuclear matter within the chiral SU(3) model in ref. [28] and in the isospin asymmetric nuclear matter in refs [29, 30]. In the present work, we investigate also the effects of the hyperons on the energies of the kaons and antikaons in the strange hadronic matter. The contribution from the vector interaction (Weinberg-Tomozawa term) leads to a drop for the antikaon energy, whereas



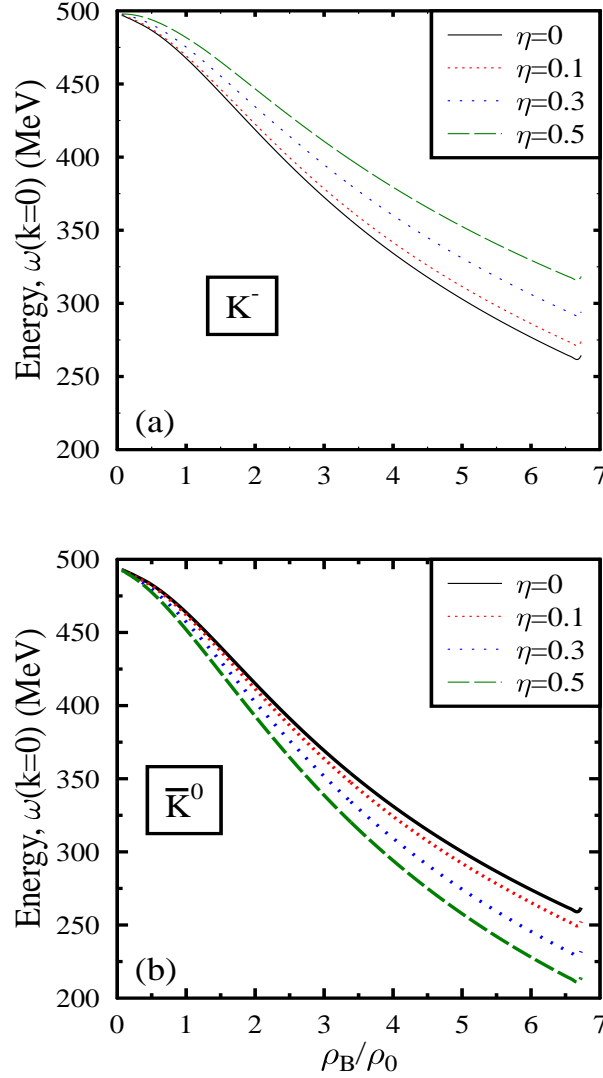


FIG. 6: The energies of the antikaons (for  $K^-$  in (a) and for  $\bar{K}^0$  in (b)), at zero momentum as functions of the baryon density ( $\rho_B/\rho_0$ ), are plotted for  $f_s=0.5$  and for different values of the isospin asymmetry parameter,  $\eta$ .

they are repulsive for the kaons in the asymmetric nuclear medium. There are now contributions from hyperons due to this vectorial interaction as given in the first term in equation (2). One may note that there is no contribution arising from either  $\Lambda^0$  or the  $\Sigma^0$ -hyperon, for such a baryon-pseudoscalar meson vectorial interaction. The scalar meson exchange term arising from the scalar-isoscalar fields ( $\sigma$  and  $\zeta$ ) is attractive for both  $K$  and  $\bar{K}$  doublets. The first term of the range term of eq. (2) is repulsive, whereas the second term has an isospin symmetric attractive contribution for both kaons and antikaons, with contributions arising due to interactions with the baryon octet. The third term of the range term has an isospin dependence [30] which now takes into account the effects of hyperons, in addition to contributions from the nucleons.

The isospin asymmetries within the kaon doublet ( $K^+$ ,  $K^0$ ) as well as in antikaon ( $K^-$ ,  $\bar{K}^0$ ) energies arise from the Weinberg-Tomazawa term due to asymmetry in proton and neutron number densities as well as asymmetries in the hyperon number densities. They also arise from the scalar-isovector  $\delta$  field becoming nonzero for the isospin

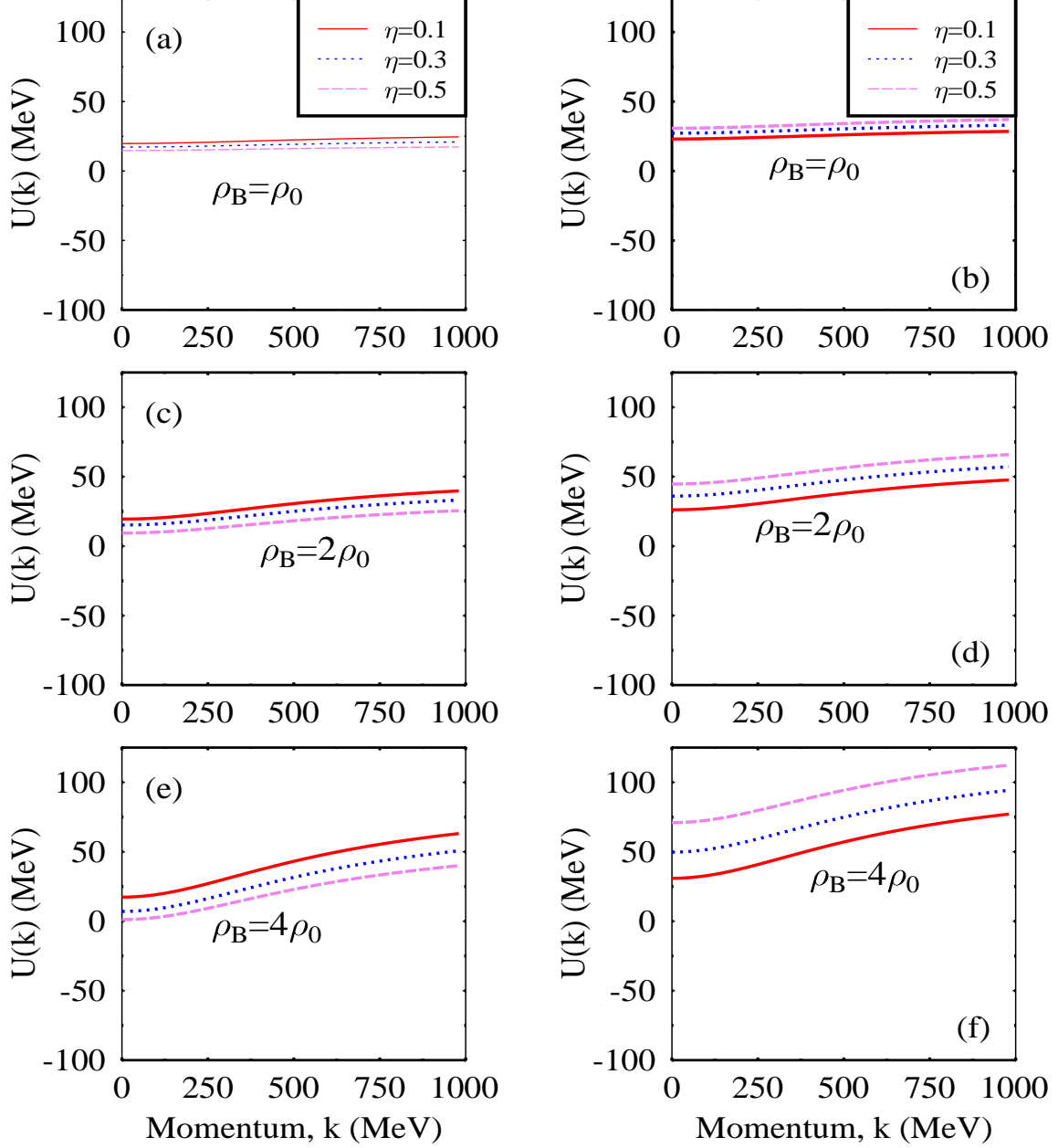


FIG. 7: The kaon optical potentials (for  $K^+$  in (a), (c) and (e) and for  $K^0$  in (b), (d) and (f)) in MeV for  $f_s=0.1$ , plotted as functions of the momentum for various baryon densities,  $\rho_B$  and for different values of the isospin asymmetry parameter,  $\eta$ .

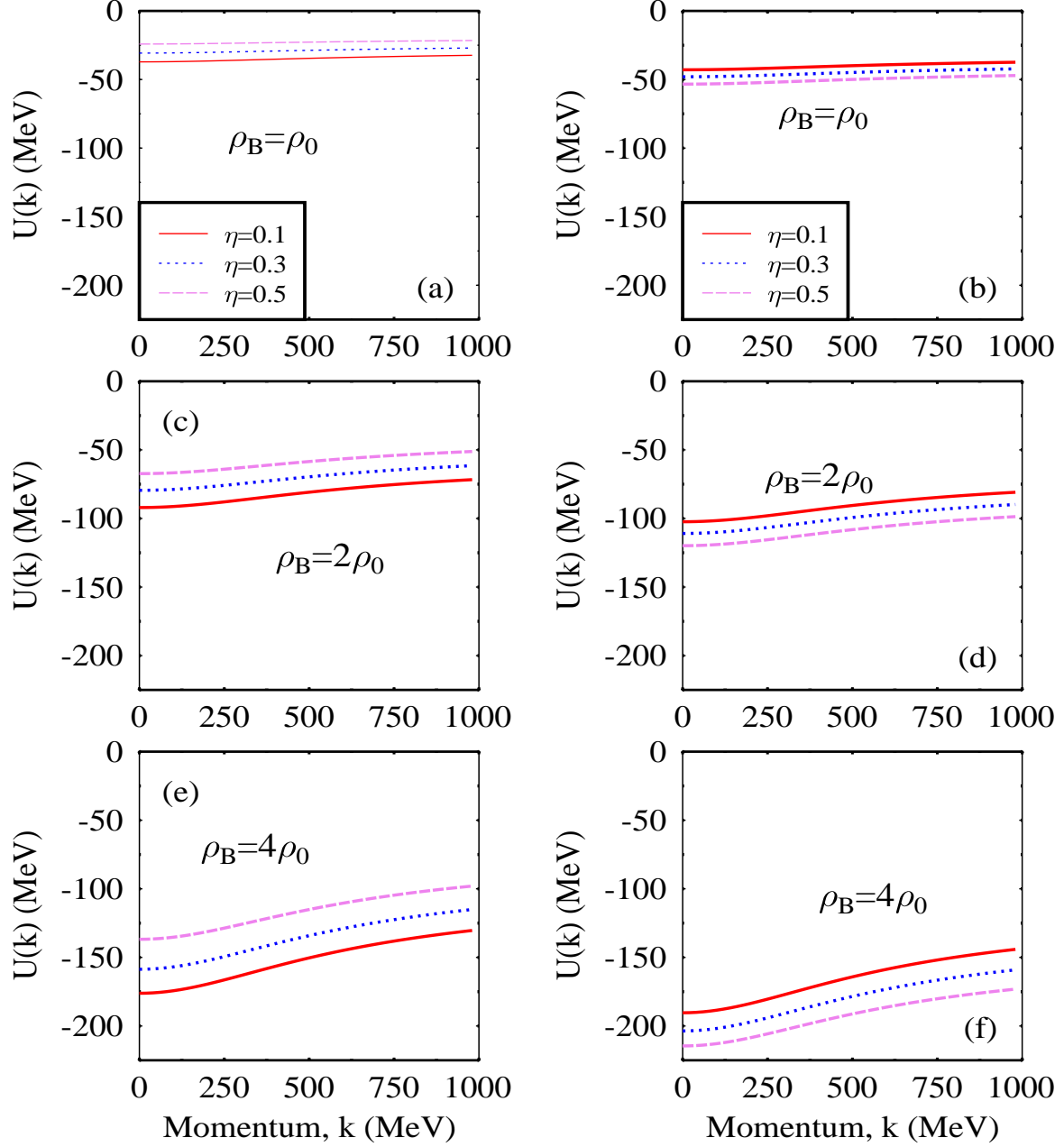


FIG. 8: The antikaon optical potentials (for  $K^-$  in (a), (c) and (e) and for  $\bar{K}^0$  in (b), (d) and (f)) in MeV for  $f_s=0.1$ , plotted as functions of the momentum for various baryon densities,  $\rho_B$  and for different values of the isospin asymmetry parameter,  $\eta$ .

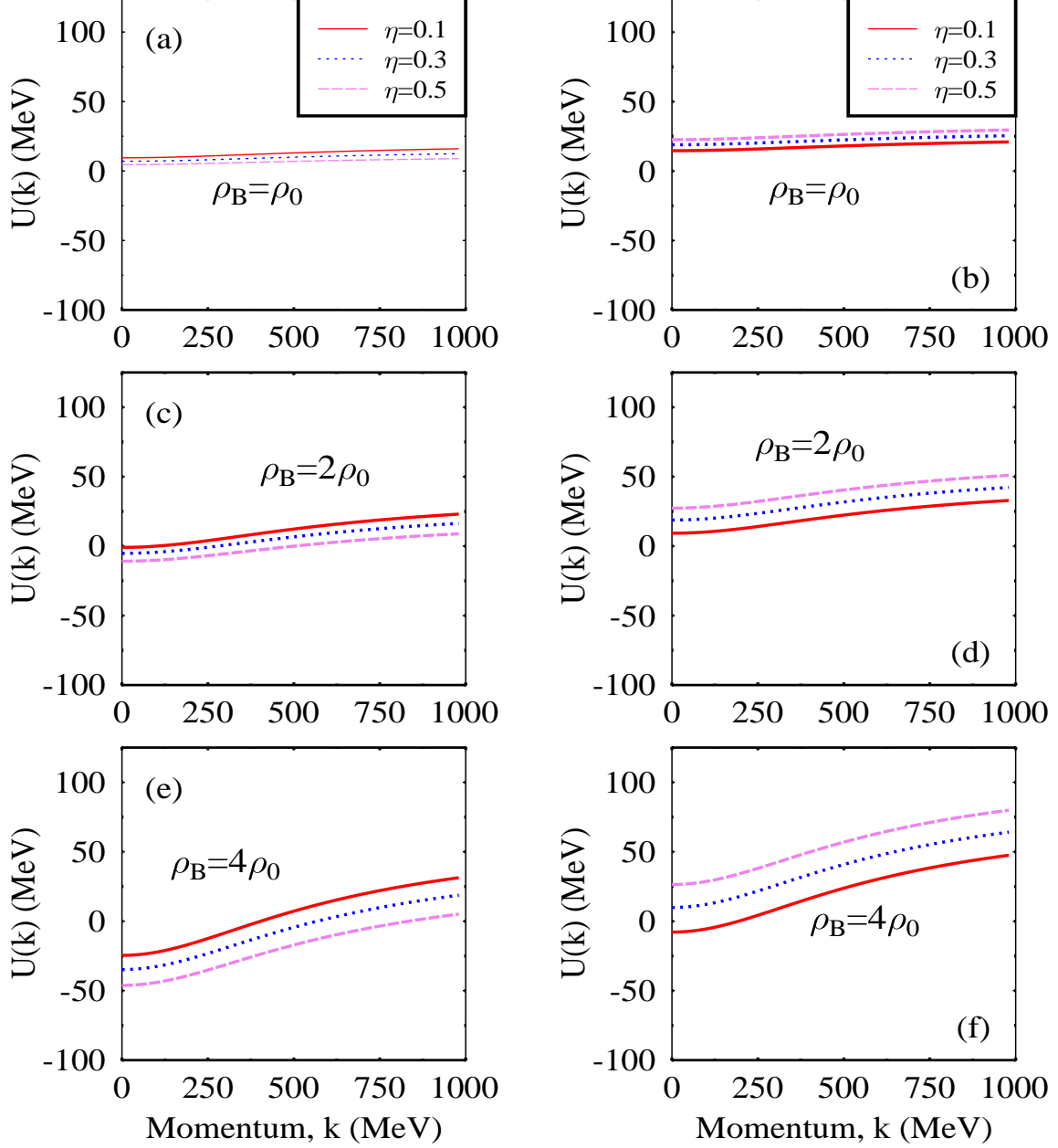


FIG. 9: The kaon optical potentials (for  $K^+$  in (a), (c) and (e) and for  $K^0$  in (b), (d) and (f)) in MeV for  $f_s=0.3$ , plotted as functions of the momentum for various baryon densities,  $\rho_B$  and for different values of the isospin asymmetry parameter,  $\eta$ .

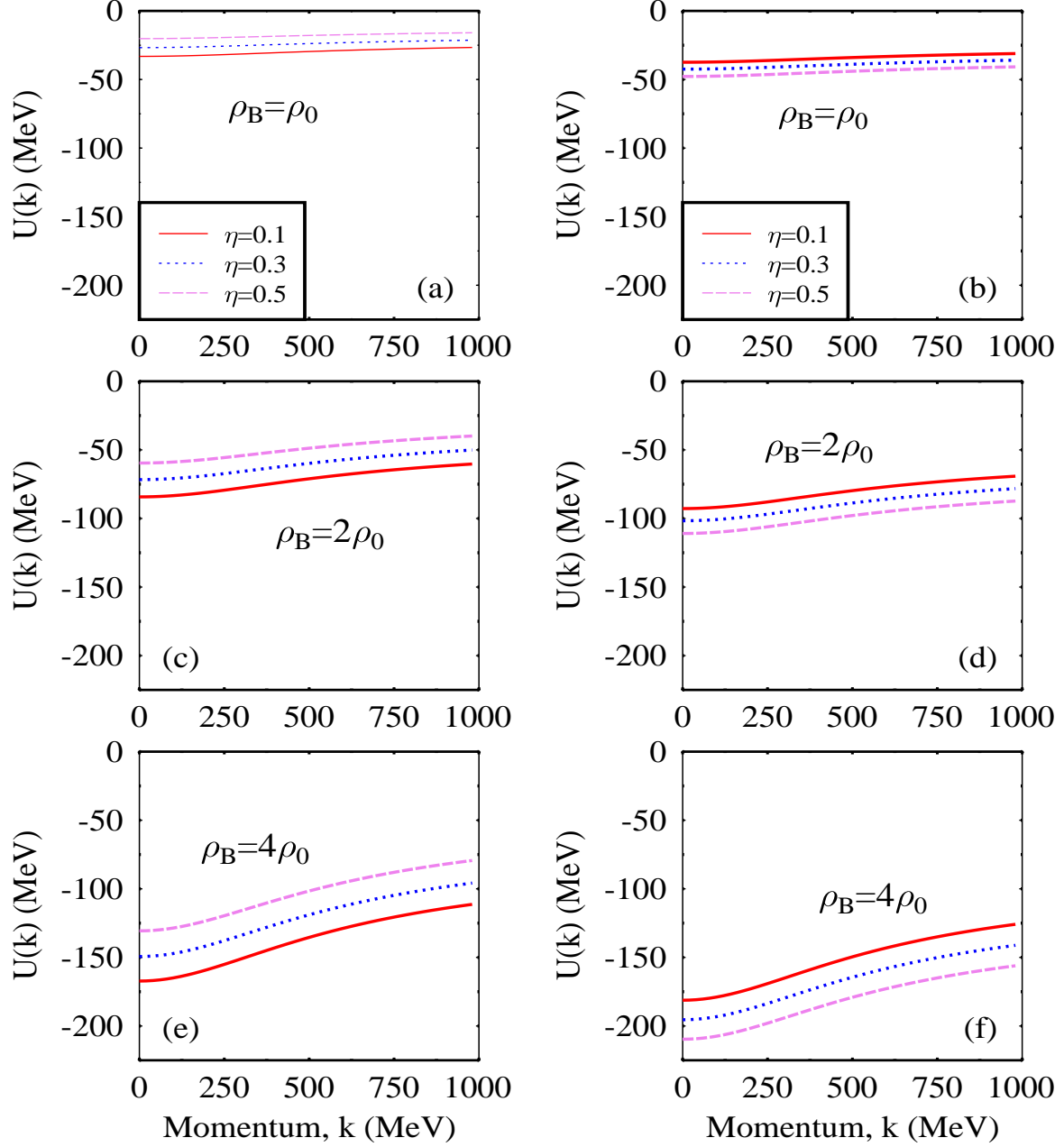


FIG. 10: The antikaon optical potentials (for  $K^-$  in (a), (c) and (e) and for  $\bar{K}^0$  in (b), (d) and (f)) in MeV for  $f_s=0.3$ , plotted as functions of the momentum for various baryon densities,  $\rho_B$  and for different values of the isospin asymmetry parameter,  $\eta$ .

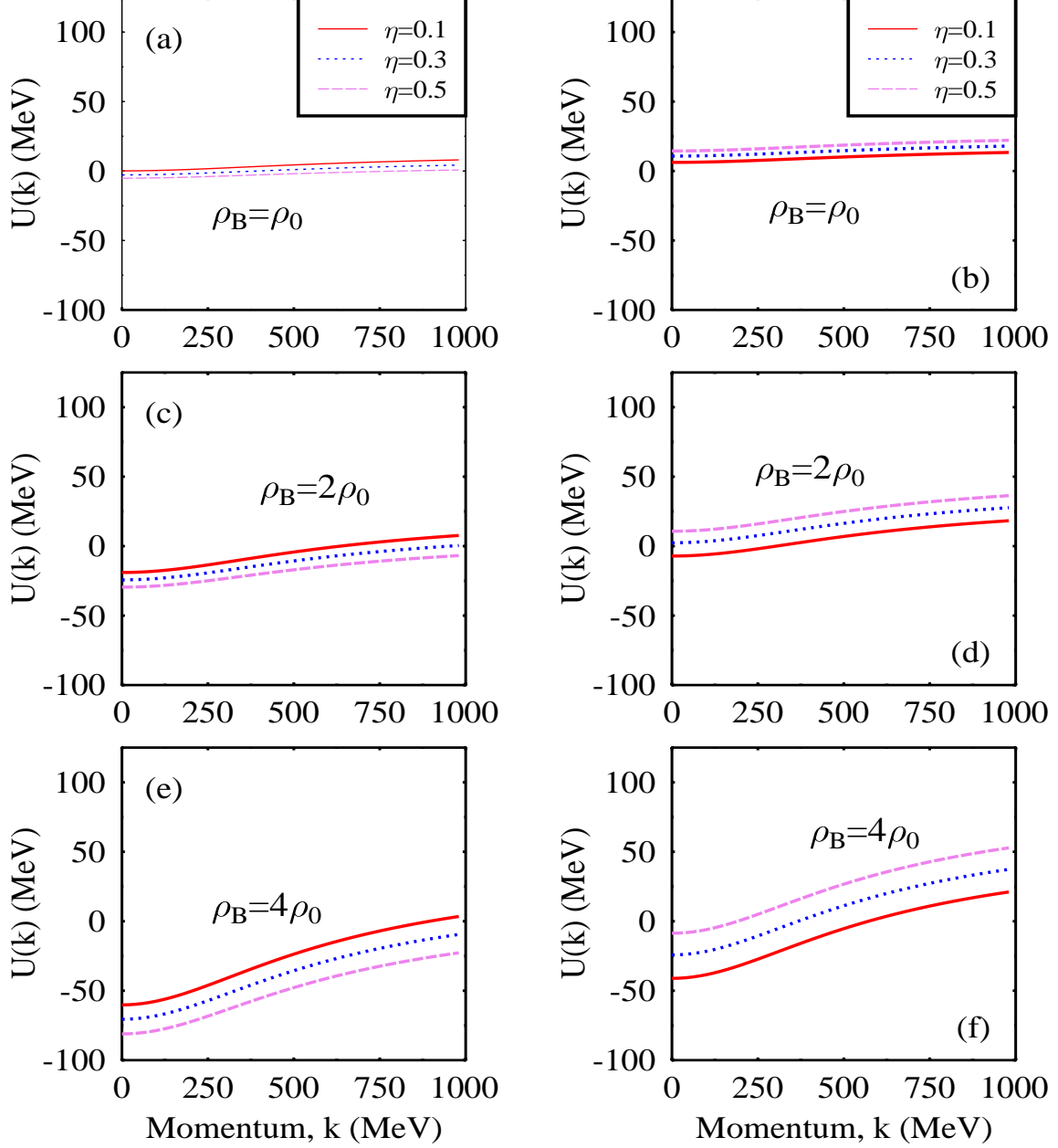


FIG. 11: The kaon optical potentials (for  $K^+$  in (a), (c) and (e) and for  $K^0$  in (b), (d) and (f)) in MeV for  $f_s=0.5$ , plotted as functions of the momentum for various baryon densities,  $\rho_B$  and for different values of the isospin asymmetry parameter,  $\eta$ .

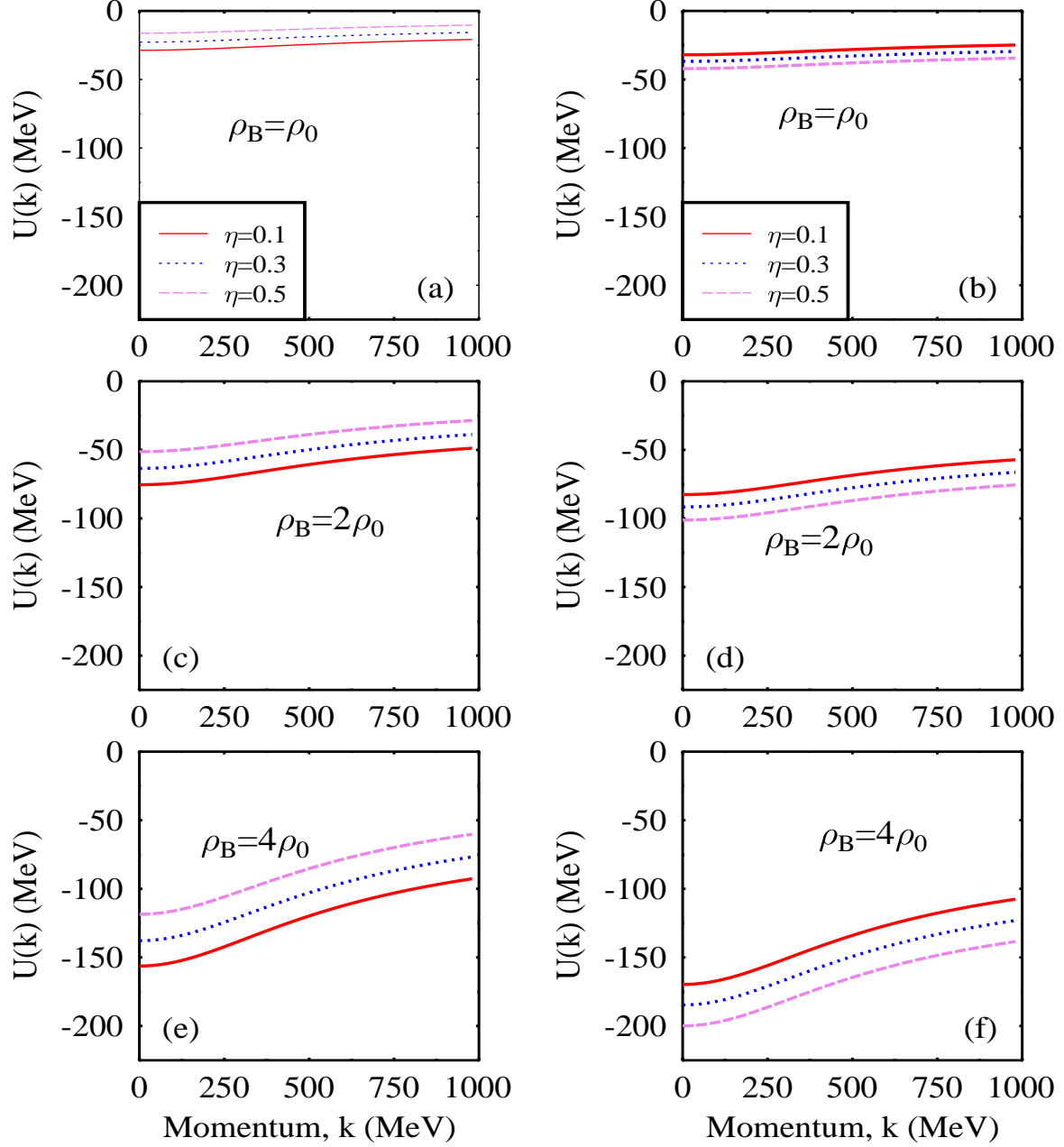


FIG. 12: The antikaon optical potentials (for  $K^-$  in (a), (c) and (e) and for  $\bar{K}^0$  in (b), (d) and (f)) in MeV for  $f_s=0.5$ , plotted as functions of the momentum for various baryon densities,  $\rho_B$  and for different values of the isospin asymmetry parameter,  $\eta$ .

asymmetric hadronic matter as can be seen from the second term (the scalar exchange term) as well as the in the range term given by the third term in equation (2). The  $d_2$  term in the interaction Lagrangian given by equation (2) also introduces asymmetries for  $K^+$  and  $K^0$ , as well as for antikaon ( $K^-$  and  $\bar{K}^0$ ) -energies, arising from both the nucleon as well as hyperonic sectors. For  $\rho_n > \rho_p$ , in the kaon sector,  $K^+$  ( $K^0$ ) has negative (positive) contributions from  $\delta$ . The  $\delta$  contribution from the scalar exchange term is positive (negative) for  $K^+$  ( $K^0$ ), whereas that arising from the range term has the opposite sign and dominates over the former contribution.

In figures 1, 3 and 5, the isospin dependent energies of the  $K^+$  and  $K^0$  at zero momentum, are plotted as functions of densities for various values of the strangeness fraction,  $f_s = \frac{\sum_i \rho_{H_i}}{2\rho_B}$ . For  $f_s=0.1$  plotted in figure 1, one sees that when the density is increased, the masses of the kaons initially increase upto about  $\rho_B = \rho_0$ . However, as the density is further increased, the kaon masses do not show a monotonic rise, but a drop at around this value. When the strangeness fraction is increased, the masses of the kaons are seen to decrease with increasing density, as one can see from figures 3 and 5, which correspond to the strangeness fractions,  $f_s=0.3$  and  $0.5$  respectively. This is due to the fact that the second term of the range term (the  $d_1$  term) which is attractive, becomes more and more dominant with increasing contributions from the hyperons, in addition to nucleons.

For  $f_s=0.1$ , at  $\rho_B = \rho_0$ , the energy of  $K^+$  is seen to drop by about 7 MeV at zero momentum when  $\eta$  changes from 0 to 0.5 whereas the  $K^0$  energy is seen to increase by about 10 MeV from the isospin zero case. The reason for the opposite behavior for the  $K^+$  and  $K^0$  on the isospin asymmetry in the nuclear medium [29, 30] originates from the vectorial (Weinberg-Tomozawa),  $\delta$  meson contribution as well as from the isospin dependent range term ( $d_2$ - term) contributions. For  $K^+$ , the  $\eta$ -dependence of the energy is seen to be less sensitive at higher densities, whereas the energy of  $K^0$  is seen to have a larger drop from the  $\eta=0$  case, as we increase the density. The isospin asymmetry in medium modifications of the  $K^+$  and  $K^0$  mesons are seen to be particularly significant for higher strangeness fractions at high densities. For  $f_s=0.5$ , at  $\rho_B = \rho_0$ , the drop (increase) in the energy of the  $K^+$  ( $K^0$ ) meson at zero momentum, from the isospin symmetric case, is about 9 MeV (11 MeV), whereas these become about 30 MeV (42 MeV) at a density of  $\rho_B = 4\rho_0$ .

For the antikaons, the  $K^-(\bar{K}^0)$  energy at zero momentum is seen to increase (drop) with the isospin asymmetry parameter,  $\eta$ . The energies of the antikaons for various values of the strangeness fraction,  $f_s$  are plotted in figures 2, 4 and 6. The sensitivity of the isospin asymmetry dependence of the energies is seen to be larger for  $K^-$  with density as compared to  $\bar{K}^0$ . The isospin asymmetry gives rise to an increase in the mass of the  $K^-$ -mesons as compared to the isospin symmetric situation of  $\eta=0$ , and hence would delay the onset on  $K^-$  condensation to higher densities, inside the neutron star matter. The energies of the antikaons as functions of densities are plotted for  $f_s=0.1$ ,  $0.3$  and  $0.5$  in figures 2, 4 and 6 respectively. The changes in the masses of the  $K^-$  and  $\bar{K}^0$  for  $f_s=0.1$ , at  $\eta=0.5$  from the isospin symmetric case, are 16 MeV (14 MeV) at  $\rho_B = \rho_0$ , and at  $\rho_B = 4\rho_0$  are 47 MeV (32 MeV). For strangeness fraction,  $f_s=0.5$ , these values are modified to 15 MeV (12 MeV) at  $\rho_B = \rho_0$  and 45 MeV (37 MeV) for  $\rho_B = 4\rho_0$ .

The qualitative behavior of the isospin asymmetry dependencies of the energies of the kaons and antikaons at finite momenta are reflected in their optical potentials plotted in figures 7, 9 and 11 for the kaons, and in figures 8, 10 and 12 for the antikaons, at selected densities, for strangeness fractions  $f_s = 0.1$ ,  $0.3$  and  $0.5$  respectively. The different behavior of the  $K^+$  and  $K^0$ , as well as for the  $K^-$  and  $\bar{K}^0$  optical potentials in the dense asymmetric nuclear matter should be observed in their production as well as propagation in isospin asymmetric heavy ion collisions. In particular an experimental study of the  $K^+/K^0$  as well as  $K^-/\bar{K}^0$  ratios as well as their collective flow in the asymmetric heavy ion collision experiments might be promising tools to investigate the isospin effects discussed here. The effects of the isospin asymmetric optical potentials could thus be observed in nuclear collisions at the CBM experiment at the proposed project FAIR at GSI, where experiments with neutron rich beams are planned to be implemented.

## V. SUMMARY

To summarize, within a chiral SU(3) model we have investigated the density dependence of the kaon and antikaon optical potentials in asymmetric hyperonic matter, arising from the interactions with the baryon octet (originating from a vectorial Weinberg-Tomozawa interaction, an isospin symmetric range term and an isospin asymmetric range term) and due to scalar-isoscalar mesons ( $\sigma$ ,  $\zeta$ ) and scalar-isovector  $\delta$  mesons. The properties of the nucleons, hyperons and scalar mesons are modified in the hadronic medium and hence due to their interactions with the kaons and antikaons, modify the  $K(\bar{K})$ -meson properties. The model with parameters fitted to reproduce the properties of hadron masses in vacuum, nuclear matter saturation properties and low energy KN scattering data, takes into account all terms up to the next to leading order arising in chiral perturbative expansion for the interactions of  $K(\bar{K})$ -mesons with baryons.



There is a significant density dependence of the isospin asymmetry on the optical potentials of the kaons and antikaons. This dependence seems to be even more dominant for larger values of the strangeness fractions in the dense hadronic matter. The results can be used in heavy-ion simulations that include mean fields for the propagation of mesons [40]. The different potentials of kaons and antikaons can be particularly relevant for neutron-rich heavy-ion beams at the CBM experiment at the future project FAIR at GSI, Germany, as well as at the experiments at the proposed Rare Isotope Accelerator (RIA) laboratory, USA. The  $K^+/K^0$  as well as  $K^-/\bar{K}^0$  ratios as well their flow pattern for different isospin of projectile and target is a promising observable to study these effects. Furthermore, the medium modification of antikaons due to isospin asymmetry in dense matter can have important consequences, for example on the onset of antikaon condensation in the bulk charge neutral matter in neutron stars. The effects of finite temperatures on optical potentials of kaons and antikaons and their possible implications on high energy heavy ion collision experiments are under investigation.

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